NANOSTRUCTURED SUPERCONDUCTORS

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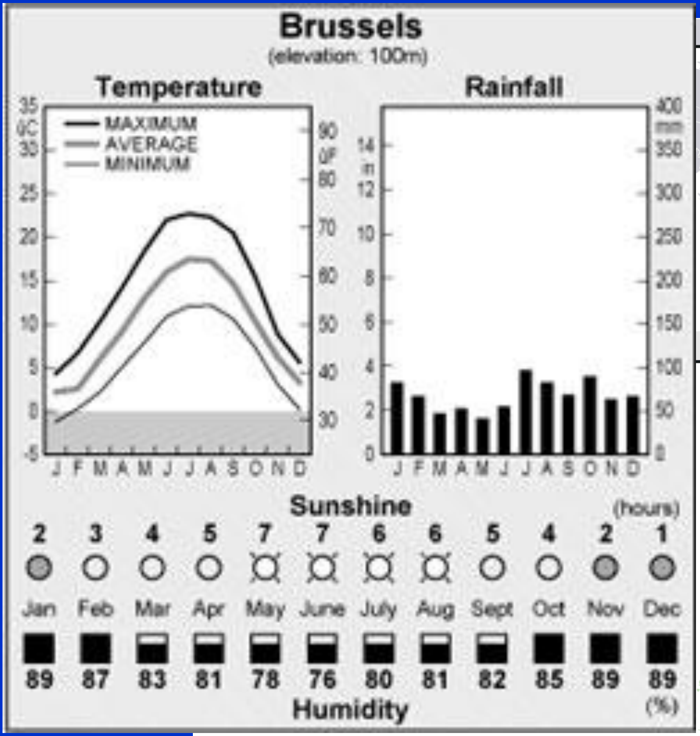
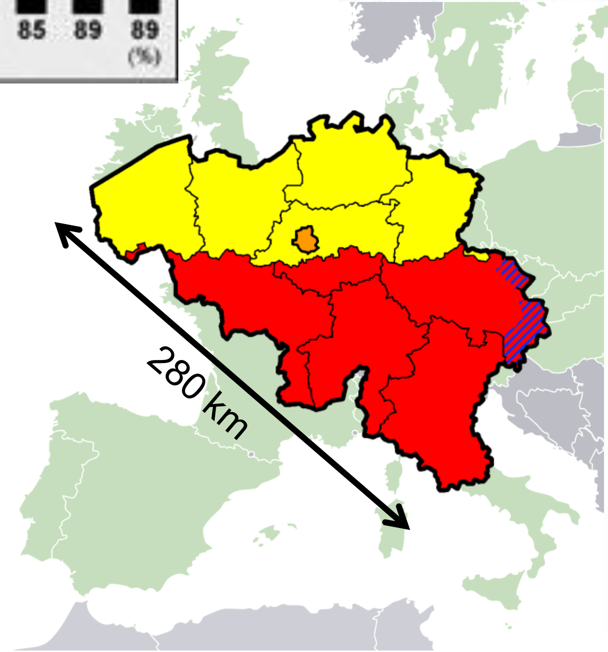




STELLA ARTOIS
ANNO 1366

Katholieke Universiteit Leuven

- Founded in 1425 by Pope Martin
- The oldest existent Catholic university in the world
- The largest University in Belgium
- 30,442 students
- 3,770 international students
- 5,109 researchers (1,276 senior and 3,833 junior researchers)

Department of Physics and Astronomy

↓

Institute of Nanoscale Physics and Chemistry (Director Victor Moshchalkov)

↓

Nanoscale Superconductivity

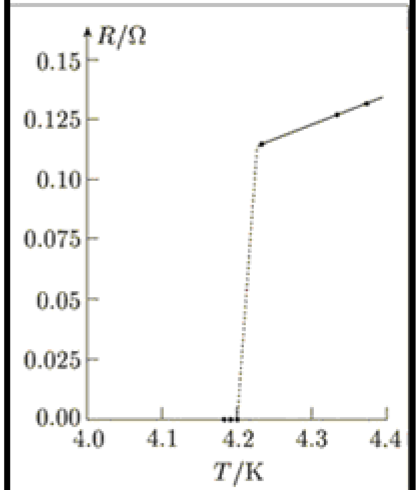
PROGRAM

- LECTURE 1 : *General Introduction to Superconductivity*
- LECTURE 2: *Mesoscopic Superconductors, Periodic Superconducting Structures and Ratchet Systems*
- LECTURE 3: *Superconductor-Ferromagnet Hybrids*

An experimental approach, so just relax !
No need to take notes !

General wisdom about Superconductivity

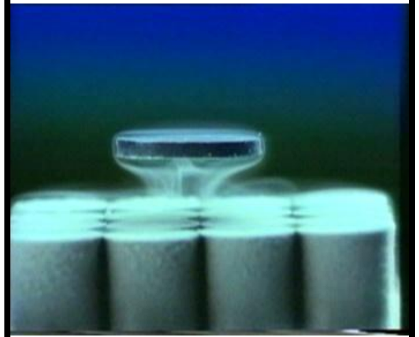
Supraconductivity
K. Onnes 1911



Perfect conductivity

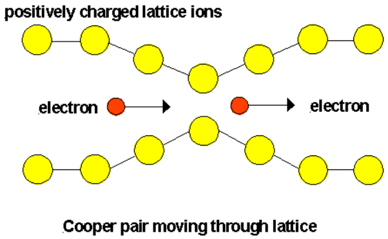
persistent currents (life time 100,000 yrs)

Meissner effect
Meissner 1933



Levitation (Violates Earnshaw theorem?)

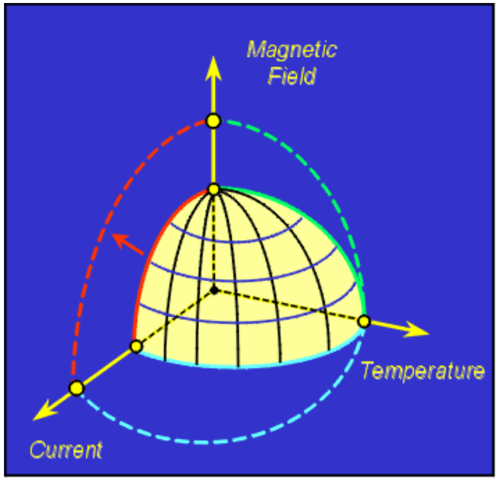
Electron pairing
Cooper 1956



Why pairing leads to no dissipation?

Can a liquid be SC?

J.E. Jaffe and N.W. Ashcroft, *Phys. Rev. B* **23**, 6176 (1981)
P. P. Edwards, C. N. R. Rao, N. Kumar, A. A. Alexandrov, *Chem.Phys.Chem.* **7**, 2015 (2006)

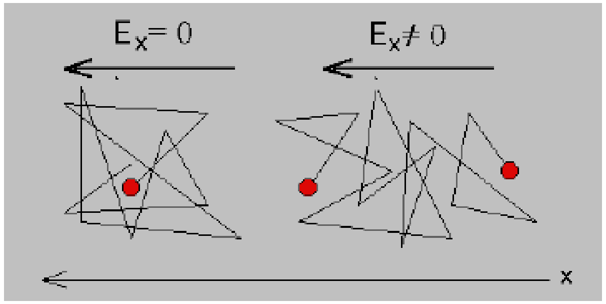


- Critical Field (Hc) → Meissner, spin flip, Clogston limit?
- Depairing Current (Jc) → self field

Normal Metals: Drude

$$F = m\ddot{x} = -eE - \eta\dot{x} \rightarrow \ddot{x} + \frac{\dot{x}}{\tau} = -\frac{eE}{m}$$

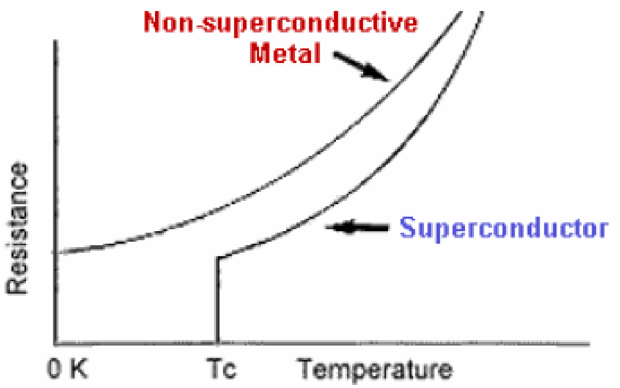
$$if \ \tau \equiv \frac{m}{\eta} \langle\langle 1 \ \rightarrow \ -eE \approx \frac{m\,\dot{x}}{\tau}$$



$$Since \ \ j = ne\dot{x} \ \ and \ \ \sigma E = j \rightarrow \sigma = \frac{ne^2\tau}{m}$$

n - density of conduction electrons, ***m*** is an effective mass of conduction electrons, and ***τ*** - average lifetime for the free motion of electrons between collision with impurities, other electrons, etc

$$\rho = \sigma^{-1} = \frac{m}{ne^2}\tau^{-1} \quad \Rightarrow$$



$$(T) \quad \sim T^2 \quad \sim T^5$$

$$\tau^{-1} = \tau_{imp}^{-1} + \tau_{el-el}^{-1} + \tau_{el-ph}^{-1}$$

$$\rho = \rho_0 + aT^2 + bT^5 + \ldots$$

Perfect conductor

$$F = m\ddot{x} = -eE - \eta\dot{x} \rightarrow \ddot{x} + \frac{\dot{x}}{\tau} = -\frac{eE}{m} \qquad if\ \tau \equiv \frac{m}{\eta} \rangle\rangle 1 \quad \rightarrow \quad -eE \approx m\ddot{x}$$

$$Since\ \ j = ne\dot{x} \rightarrow \frac{dj}{dt} = ne\ddot{x} = -\frac{ne^2}{m}E$$

A constant E produces an ever increasing j !

$$\text{Appying } \nabla\times\frac{dj}{dt} = -\frac{ne^2}{m}\nabla\times E \underset{\uparrow}{=} \frac{ne^2}{mc}\frac{\partial B}{\partial t} \underset{\uparrow}{=} \frac{ne^2}{mc}\frac{\partial\nabla\times A}{\partial t}$$

Maxwell $\qquad B \equiv \nabla\times A$

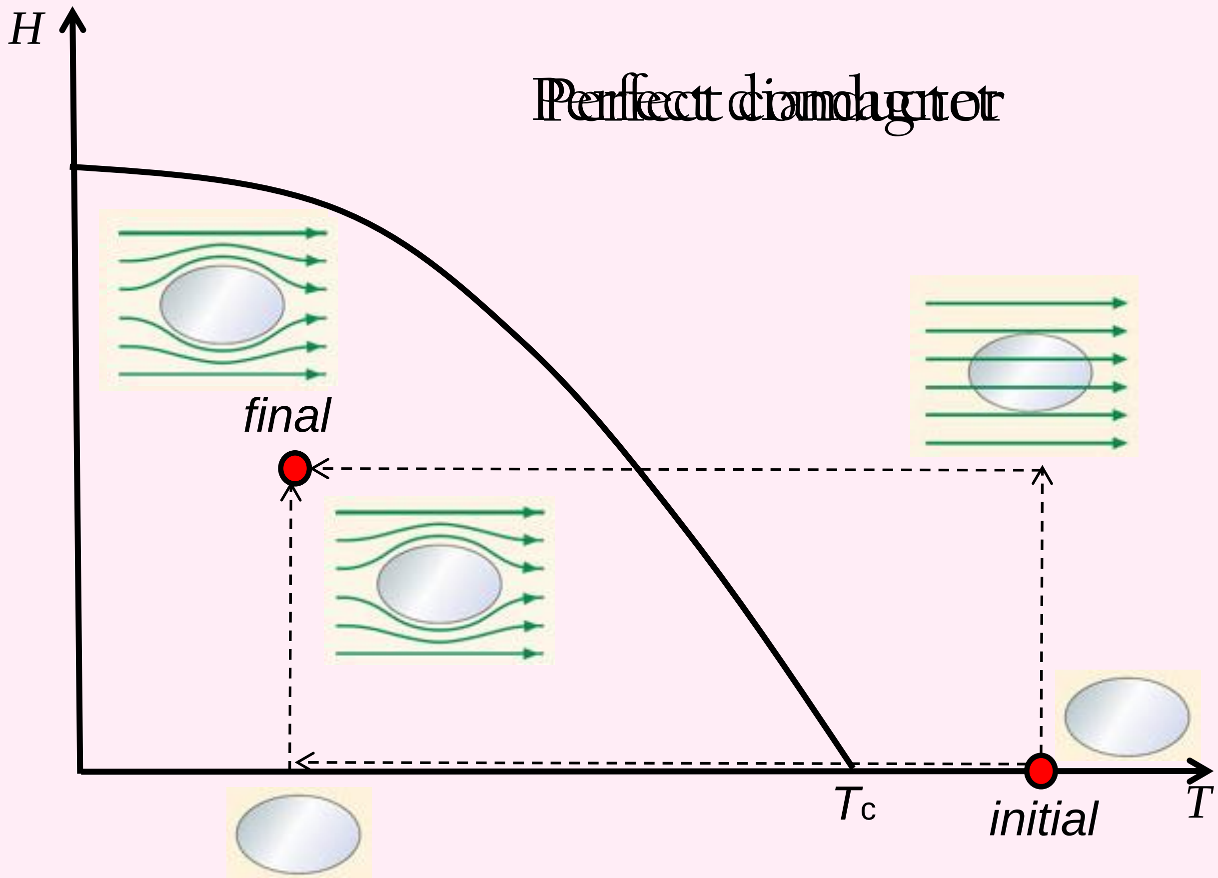
London brothers
SC is homogeneous

…LATIONS

Perfect Conductor

$J \propto A$

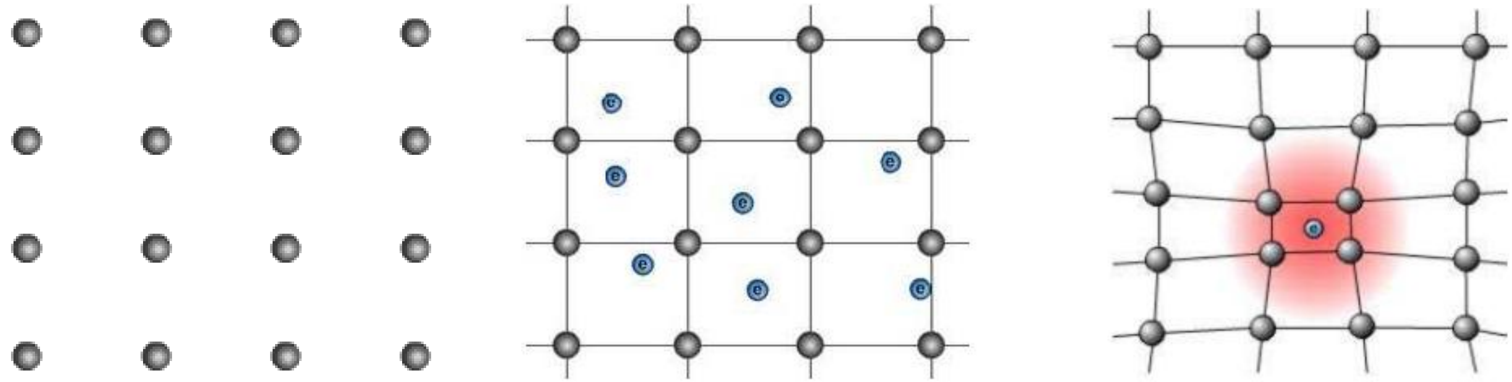
Perfect diamagnet vs perfect conductor



Why and how is the flux expelled at the transition if Faraday law applies?

Due to the conservation of the angular momentum should the SC rotate when crossing the transition ? R.H. Pry, A.L. Lathdrop, W.V. Houston, *Phys.Rev.* **86**, 905 (1952)

Microscopic picture



Bare electrons repel each other with electrostatic potential:

$$V(\mathbf{r}-\mathbf{r}') = \frac{e^2}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|} \sim 1 \text{ eV/atom}$$

$$V(\mathbf{r}-\mathbf{r}') = \frac{e^2}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|} e^{-|\mathbf{r}-\mathbf{r}'|/r_{TF}}$$

$$\frac{\text{Size of a Cooper pair}}{r_{TF}} \approx \frac{\varepsilon_F}{\Delta} \sim 10^4$$

Effective interaction of electrons due to exchange of phonons

Frolich (1950)

$$V_{eff}(\omega) = |g_{eff}|^2 \frac{1}{\omega^2 - \omega_D^2}$$

Frequency dependence reflecting
The retarded nature of the interaction

- g_{eff} is an effective constant of electron-phonon interaction
- ω_D is a typical Debye phonon frequency

Retarded nature of the e-phonon interaction

What is the shortest response time of the lattice ?

⇩

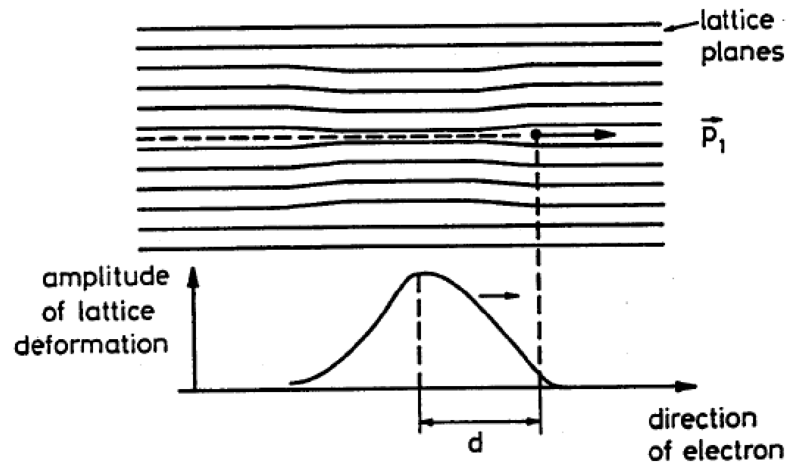
Highest possible lattice vibration = ω_D

$$\hbar\omega_D = k_B\Theta_D \approx 0.01 - 0.02 \text{ eV} .$$

⇩

The maximum lattice deformation lags behind the electron by

$$d \approx v_F \frac{2\pi}{\omega_D} \approx 100 - 1000 \,\text{nm} .$$



This set a limit to the minimum ξ possible

⇩

set a maximum $T_c \sim \Theta_D$

How big is a Cooper pair ?
What is the total momentum of a Cooper pair ?
Is the Cooper pairing a singlet or a triplet state ?

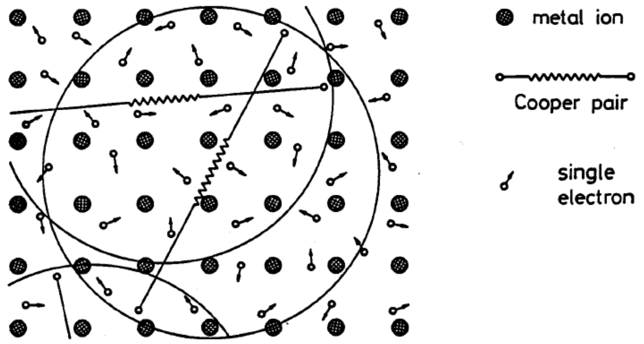
Size of the Cooper pairs

$$\Delta x \Delta p \geq \hbar$$

$$\Delta p \approx \frac{kT_c}{v_F}$$

$$\Rightarrow \quad \Delta x = \xi \approx \frac{\hbar v_F}{kT_c}$$

Factor of 5 too big



- Because they are weakly bound Cooper pairs constantly breaking up and reforming.
- The weak binding also causes them to be large.
- In the region of the pair there are many electrons that would like to bond into a pair.

Frolich (1950) – Cooper (1956) – BCS (1957)

(i) The effective forces between electrons can sometimes be **attractive** in a solid rather than repulsive

(ii) „Cooper problem" ⇒ two electrons outside of an occupied Fermi surface form a stable pair bound state, and this is true **however weak the attractive force**

(iii) Schrieffer constructed a **many-particle wave function** which all the electrons near the Fermi surface are paired up

$$E - 2E_F = \frac{-2\hbar\omega}{e^{2/N(0)V} - 1}$$

There is a gap !
$\Delta = |E - 2E_F|$

Cooper pairs can only scatter when they gain sufficient energy to cross the gap

⇩

Origin of the lack of dissipation

Analogy with atomic physics !

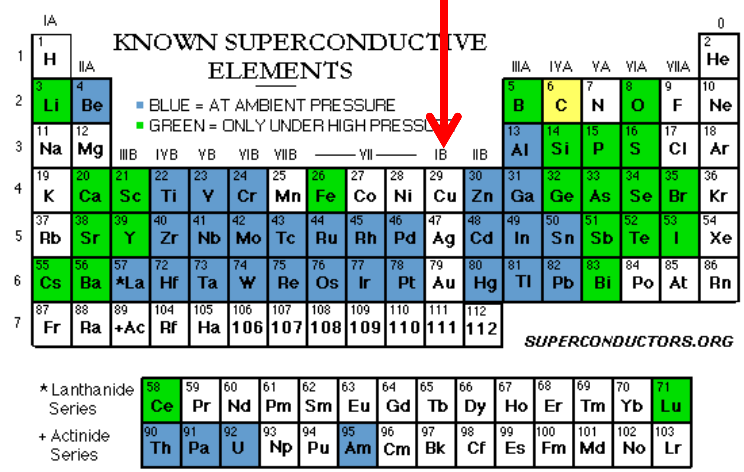
If the presence of a gap justifies the supercurrents, why a semiconductor is not a superconductor?

Good conductors vs Superconductors

Good conductors

- Silver (1.62 uOhm-CM)
- Copper (1.67)
- Gold (2.2)
- Aluminum (2.6)
- Calcium (~3.8)
- Beryllium (4.0)
- Magnesium (4.4)
- Sodium (4.7)
- Molybdenum (4.77)
- Iridium (5.3)
- Tungsten (~5.5)
- Zinc (5.9)

Superconductors



Why Ag, Cu and Au are not SC?

Not only e^- are necessary to superconduct

Effective interaction of electrons due to exchange of phonons

$$V_{eff}(\omega) = \left|g_{eff}\right|^2 \frac{1}{\omega^2 - \omega_D^2}$$

Large *g* is needed to have superc.

Strong e-phonon coupling implies high resistance at *T*room

THEORICAL APPROACHES

London theory 1935

Homogeneous superconducting state

$E = \lambda^2 \partial J/\partial t$ → Perfect conductivity

$\Delta^2 B = B/\lambda^2$ → Perfect Diamagnetism

$\lambda >> \xi$

Ginzburg-Landau 1950

IRRESPECTIVE OF THE MICROSCOPIC MECHANISM DRIVING THE PHASE TRANSITION, THEY CAN BE CLASSIFIED ACCORDING TO THEIR COMMON BEHAVIOR

Gork'ov 1959

BCS theory 1957

Eliashberg Theory → Extension of BCS to strong coupling

G.M. Eliashberg, Sov. Phys. JETP 3 696 (1963)

Usadel → Simplification of Eliashberg theory for dirty SC

K.D. Usadel, Phys. Rev. Lett. 25, 507 (1970)

Epistemology

GENERAL ARTICLES CURRENT SCIENCE, VOL. 94, NO. 10, 25 MAY 2008

Phenomenologism vs fundamentalism: The case of superconductivity

Towfic L. Shomar

Towfic L. Shomar is in the Department of Human and Social Sciences, Philadelphia University, Jordan and the Centre for the Philosophy of Human and Social Science, London School of Economics, London.

This article argues that phenomenological treatment of physical problems is more powerful than fundamental treatment. Developments in the field of superconductivity present us with a clear example of such superiority. The BCS (Bardeen, Cooper and Schrieffer) was accepted as the fundamental theory of superconductivity for a long time. Nevertheless, Landau and Ginzburg phenomenological model has so far proven to be a more fruitful theoretical representation to understand and to predict the features of superconductivity and superconductive materials.

Why to focus in thermodinamics if we already have a microscopic theory ?

How can you understand the exact properties of superconductors, like exactly zero resistance and exact flux quantization, on the basis of an approximate dynamical theory (BCS)? It is only the argument from exact symmetry principles that can fully explain the remarkable exact properties of superconductors.

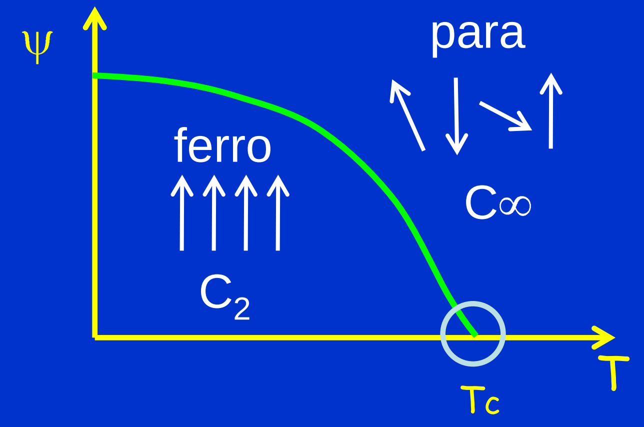
All of the dramatic exact properties of superconductors follow from the assumption that electromagnetic gauge invariance is broken in this way, with no need to inquire into the mechanism by which the symmetry is broken.

Steven Weinberg
AAPPS Bulletin April 2008, Vol. 18, No. 2

CONTINUOUS TRANSITIONS

Breaking symmetry ⟷ Order Parameter ψ

LOWER SYMMETRY
HIGHER ORDER



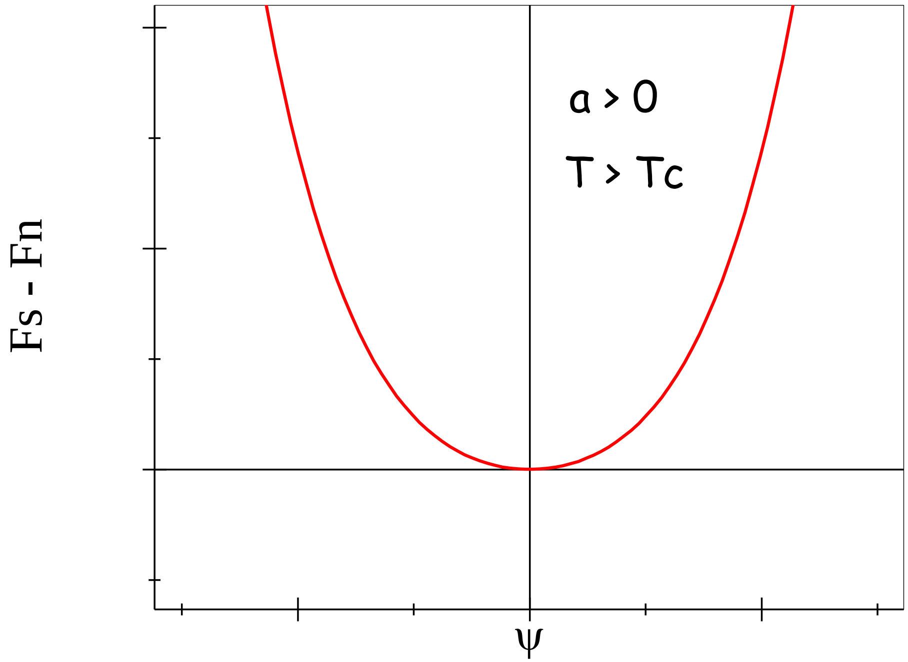
$\psi(T \sim T_c) \sim 0$ ⟶ $F = F(T>T_c) + a\psi + b\psi^2 + c\psi^3 + d\psi^4 + \ldots$

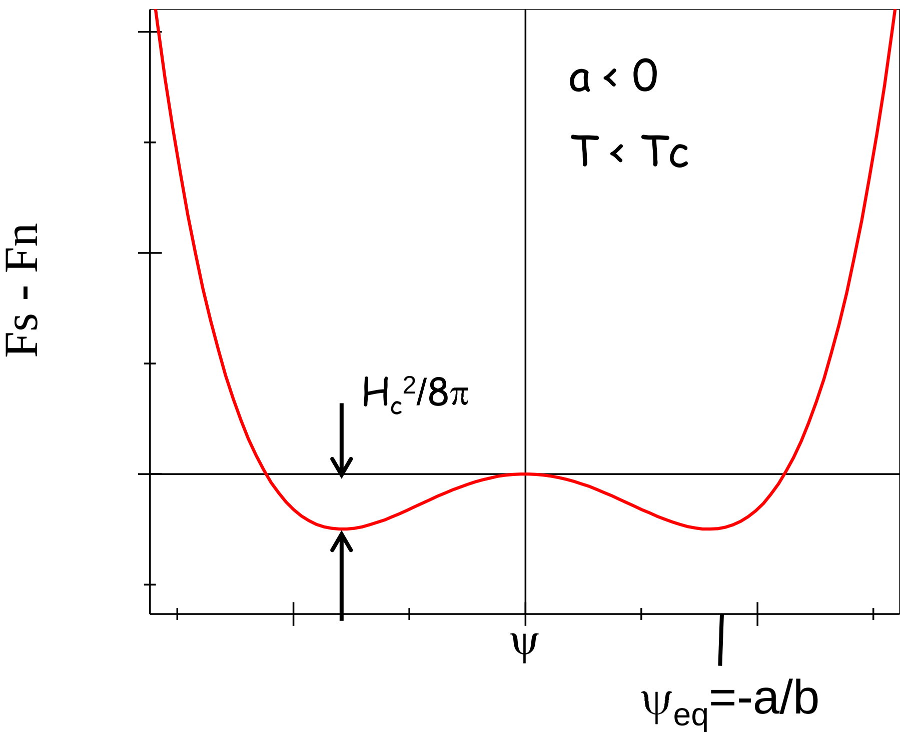
SC → Ψ complex   Magnetism → Ψ vector   F must be real

F should be differentiable (analytic) around $\psi \sim 0$

$$F = F(T>T_c) + a|\psi|^2 + \tfrac{1}{2}b|\psi|^4 + \ldots$$

$$F = F(T>T_c) + a|\psi|^2 + \tfrac{1}{2}b|\psi|^4 + \ldots$$





$a \sim a_o(T-T_c)/T_c$

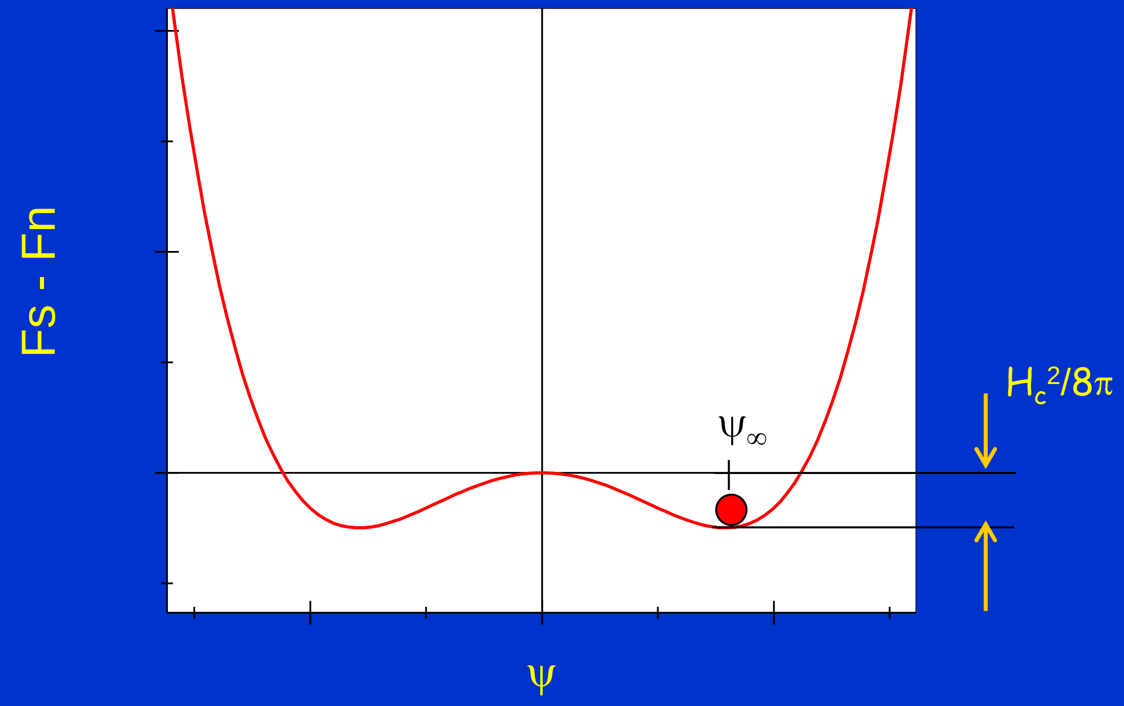
Superconductivity → Wave function

$$\psi = \psi_0 e^{-i\varphi}$$

The phase φ quenches at the transition

$n_s = \psi^* \psi$

By definition GL should be valid near Tc… but how close?



The mean field theory is valid as long as $kT < H_c^2/8\pi \times \xi^3$

Ginzburg-Levanyuk criterion

FULL GL FOR A SUPERCONDUCTOR

Inhomogeneous system $\psi = \psi(\mathbf{r})$

punish rapid changes in n_s $h|\nabla|\psi(\mathbf{r})||^2/2m$ Dislike N-S domain boundaries

Magnetic fields

$$\frac{\hbar}{i}\boldsymbol{\nabla} \rightarrow \frac{\hbar}{i}\boldsymbol{\nabla} - q\mathbf{A}$$

Supercurrent kinetic energy $n_s\, p^2/2m = (h\nabla\varphi - eA/c)^2|\psi|^2/2m$

$$F = F_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}\left|(-i\hbar\nabla - 2e\mathbf{A})\psi\right|^2 + \frac{|\mathbf{H}|^2}{2\mu_0}$$

Minimizing F with respect to ψ and A

$$\begin{cases} \alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\mathbf{A})^2\psi = 0 \\ J = \frac{e\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{2e^2}{mc}\psi^*\psi A \end{cases}$$

Superconductivity, Superfluids and Condensates, James F. Annett (Oxford Series)

COHERENCE LENGTH

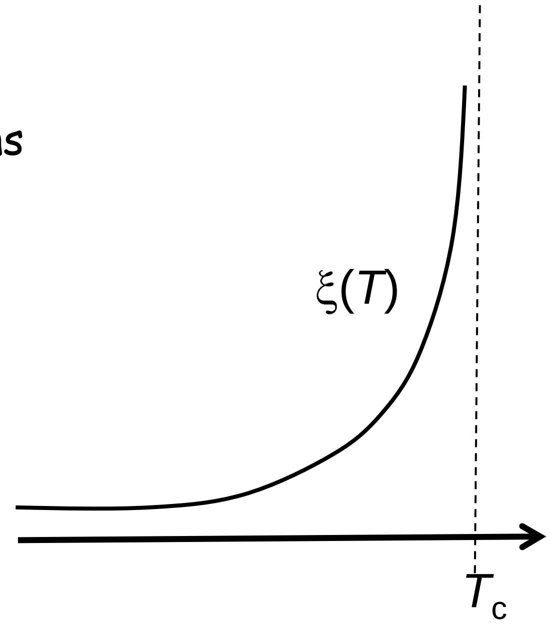
$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}\left(-i\hbar\nabla - 2e\mathbf{A}\right)^2\psi = 0$$

No field applied → A=0 → eq. with real coeff. → ψ real

$$\alpha\psi + \beta\psi^3 + \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial r^2} = 0$$

Characteristic scale for ψ variations

$$\xi^2(T) = -\frac{\hbar^2}{2m\alpha} \approx \left(1 - \frac{T}{T_c}\right)^{-1}$$



Does ξ depends on field ?

PENETRATION DEPTH

$$J = \frac{e\hbar}{2mi}\left(\psi^{*}\nabla\psi - \psi\nabla\psi^{*}\right) - \frac{2e^{2}}{mc}\psi^{*}\psi A$$

Rigid wave function (London) → $\psi = \psi_{\infty}$

$$J = -\frac{2e^{2}}{mc} n_{s} A$$

$$\nabla \times J = -\frac{2e^{2}}{mc} n_{s} B$$

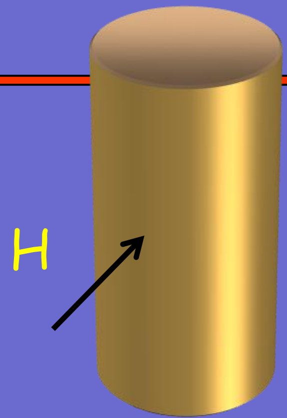
$$\nabla \times \nabla \times B = -\frac{2e^{2}}{mc} n_{s} B$$

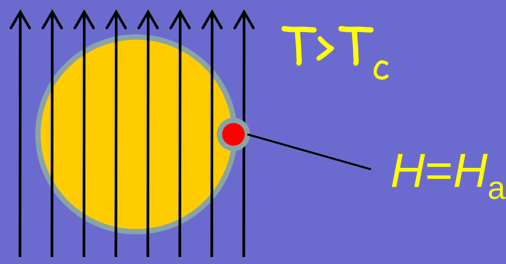
$\lambda(T)^2 \sim 1/(1-T/T_c)$

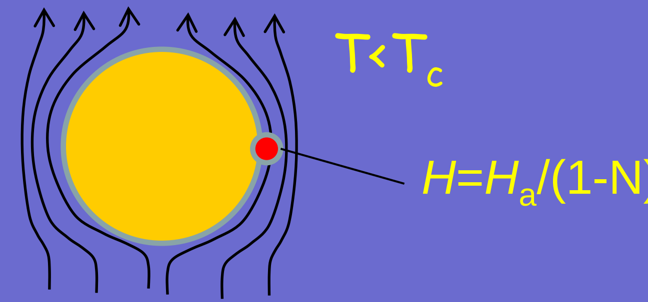
$$\frac{mc}{2e^{2}n_{s}}\nabla^{2}B = -B$$

$\kappa = \lambda(T) / \xi(T)$ = constant

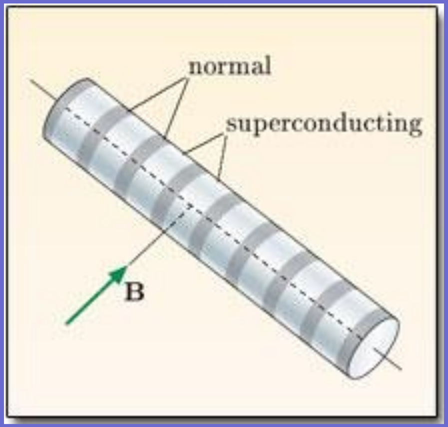
Coexisting N and S states

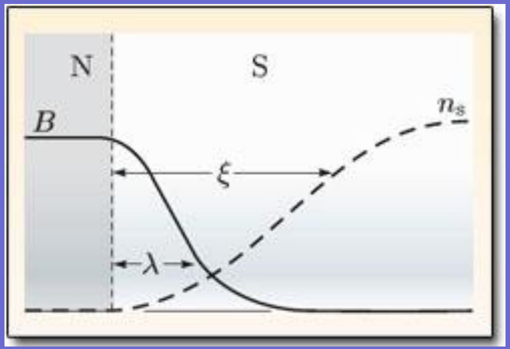






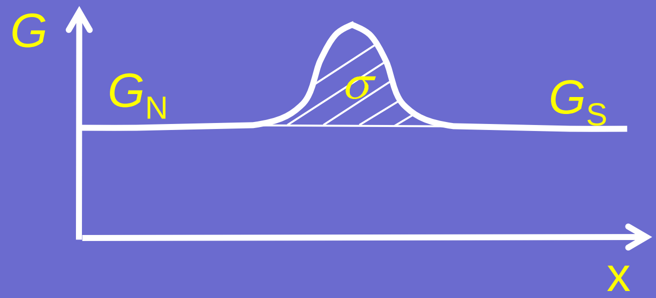
Sphere → $N=1/3$
Cylinder → $N=1/2$





Coexistance of SC and NM → $H=H_c$

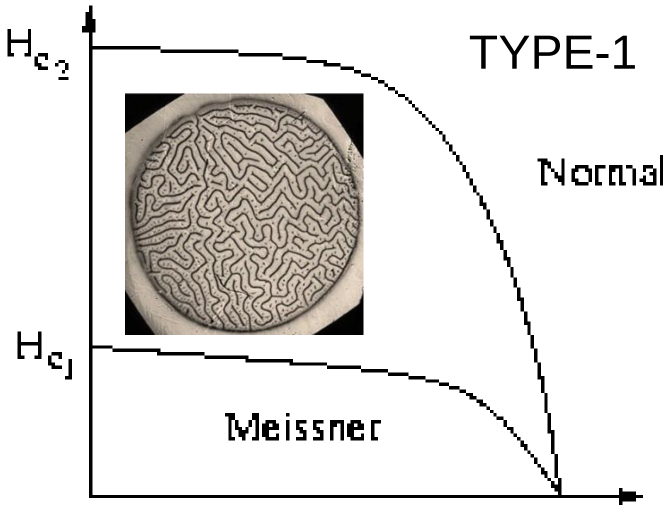
What is the cost to build a N-SC boundary?

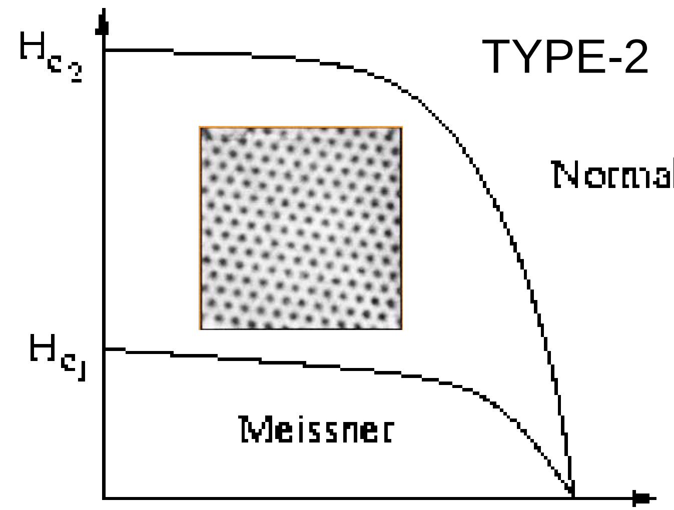


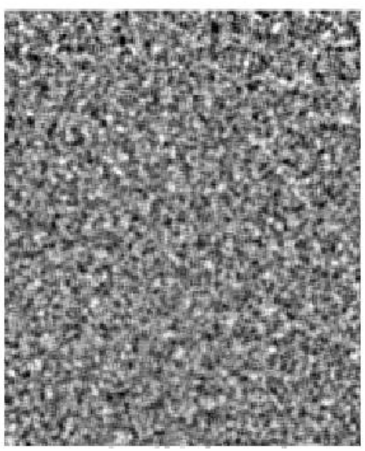
Gaining condensation energy (ξ scale)
Loosing energy of flux expulsion (λ scale)

$$\sigma \propto \frac{H_c^2}{8\pi}(\xi - \lambda)$$

Two types of flux penetration







H = 369 G

size › 20 μm

size ‹ 0.2 μm

White stripes → SC state

Two types of superconductors

Type I → $\kappa = \lambda/\xi < 1/\sqrt{2}$

Lead (Pb)	7.196 K
Lanthanum (La)	4.88 K
Tantalum (Ta)	4.47 K
Mercury (Hg)	4.15 K
Tin (Sn)	3.72 K
Indium (In)	3.41 K
Palladium (Pd)*	3.3 K
Chromium (Cr)*	3 K
Thallium (Tl)	2.38 K
Rhenium (Re)	1.697 K
Protactinium (Pa)	1.40 K
Thorium (Th)	1.38 K
Aluminum (Al)	1.175 K
Gallium (Ga)	1.083 K
Molybdenum (Mo)	0.915 K
Zinc (Zn)	0.85 K
Osmium (Os)	0.66 K
Zirconium (Zr)	0.61 K
Americium (Am)	0.60 K
Cadmium (Cd)	0.517 K
Ruthenium (Ru)	0.49 K
Titanium (Ti)	0.40 K
Uranium (U)	0.20 K
Hafnium (Hf)	0.128 K
Iridium (Ir)	0.1125 K
Beryllium (Be)	0.023 0.0154 K
Tungsten (W)	0.0019 K
Platinum (Pt)*	0.000325 K

Type II → $\kappa = \lambda/\xi > 1/\sqrt{2}$

Nb	9.25 K
Tc	7.8 K
V	5.4 K

- Higher Tc !

$\xi_o T_c$ ~ constant

Is it possible to switch from type I to type II?

Electronic mean free path
(due to non-local electrodynamics)

$\xi(0) \sim 0.855\sqrt{(\xi_o \ell)}$

$\lambda(0) \sim 0.64\lambda_L\sqrt{(\xi_o/\ell)}$

Thickness dependence of λ
(due to electromagnetism in 2D)

$\Lambda \sim \lambda^2/t$

Is it possible to switch from type II to type I?

LINEAR GL EQUATIONS

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}\left(-i\hbar\nabla - 2e\mathbf{A}\right)^2\psi = 0$$

$$J = \frac{e\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{2e^2}{mc}\psi^*\psi A$$

Near Tc → $|\psi| \ll |\psi_\infty|$ ⇒ $|\psi| \gg |\psi|^2\psi$ ⇒ $\left(\nabla - i\frac{2e}{\hbar}A\right)^2\psi = \frac{\psi}{\xi^2}$

Since $J \sim |\psi|^2$ → $A = A_{appl}$ ⟶ Eq. for J and eq. for $|\psi|$ are decoupled !

BULK NUCLEATION OF SC: Hc2

LINEAR GINZBURG-LANDAU EQUATION

$$\frac{\hbar^2}{2m}\left(\nabla - i\frac{2e}{\hbar}A\right)^2\psi = \frac{\hbar^2}{2m\xi^2}\psi$$

$$\varepsilon_n = \frac{\hbar^2}{2m\xi_n^{\ 2}}$$

SCHRODINGER EQUATION

$$\hat{H}\psi_n = \varepsilon_n\psi_n$$

$$\hat{H} = \frac{1}{2m}\left(\hat{p} - q\hat{A}\right)^2$$

$$\hat{p} = -i\hbar\nabla$$

$$\varepsilon_n = \frac{q\hbar H}{m}\left(n + \frac{1}{2}\right)$$

The lowest energy state corresponds to *n*=0

$$H_{\max} = \frac{\hbar}{2e\xi^2}$$

Maximum possible field

BULK NUCLEATION OF SC: H_{c2}

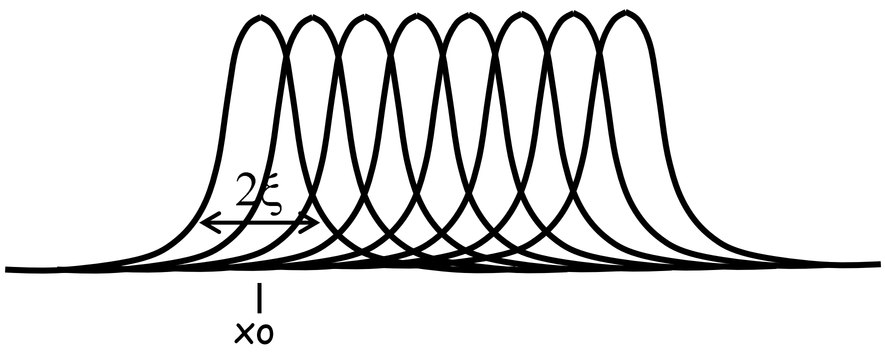
EIGENVALUE

$$H_{\max} = \frac{\hbar}{2e\xi^2}$$

EIGENFUNCTION

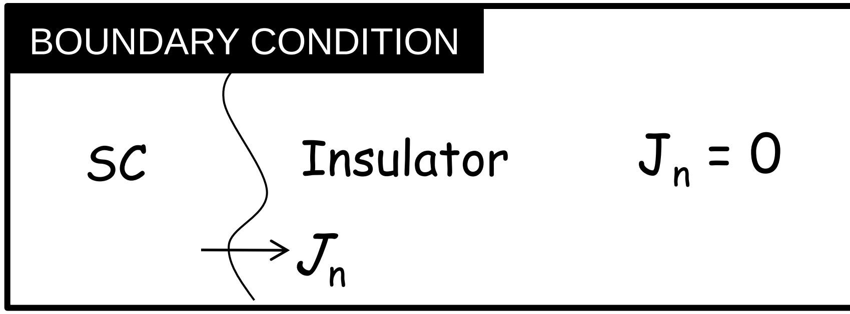
For $H//z$

$$\psi = e^{ik_y y} e^{ik_z z} \exp\left(-\frac{(x-x_0)^2}{2\xi^2} \right)$$



x_0 is arbitrary in bulk

Is it x_0 arbitrary if we have a boundary ?



$$\mathbf{J} = \frac{2e}{m}\left(\psi^*\left(-i\hbar\nabla - 2e\mathbf{A}\right)\psi\right)$$

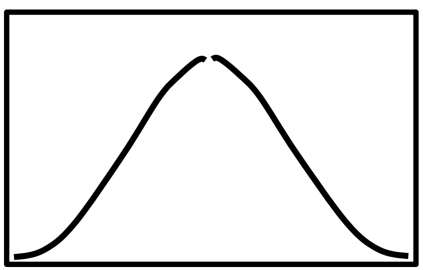
Dirichlet boundary condition

$$\psi = 0$$

Neumann boundary condition

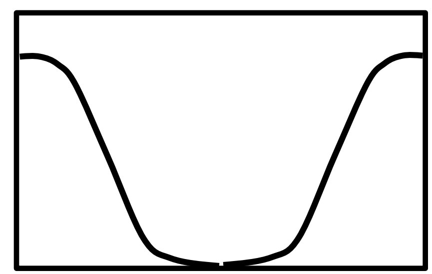
$$(-i\hbar\nabla - 2eA)\psi\Big|_{\perp} = 0$$

Schrödinger equation



Particle wavefunction $|\Psi|^2$

Ginzburg-Landau equation



Complex order parameter $|\Psi|^2$

A LOWER EIGENVALUE CAN BE FOUND IF $x_0 \sim \xi$

$$H \approx 1.7 H_{max}$$

FLUXOID

$\psi = |\psi| e^{i\varphi}$ Single-valued function $\oint \nabla\varphi \cdot dl = 2\pi n$

$$J = \frac{e\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{2e^2}{mc}\psi^*\psi A \quad \Rightarrow \quad J = \frac{e}{m}|\psi|^2\left(\hbar\nabla\varphi - \frac{2e}{c}A\right) \equiv (2e)n_s v_s$$

$$\nabla\varphi = \frac{2m}{\hbar}v_s + \frac{2e}{\hbar}A$$

$$\oint \nabla\varphi \cdot dl = \frac{2e}{\hbar}\underbrace{\oint A \cdot dl}_{\Phi} + \frac{2m}{\hbar}\oint v_s \cdot dl = 2\pi n$$

$$\Phi + \frac{m}{e}\oint v_s \cdot dl = n\frac{h}{2e} = n\Phi_o = \Phi'$$

FLUX QUANTUM

$$\Phi_o = \frac{h}{2e} = 2.07\ [mT\mu m^2]$$

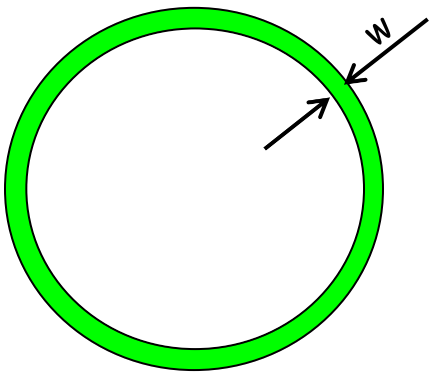
Φ' is quantized, not Φ !!

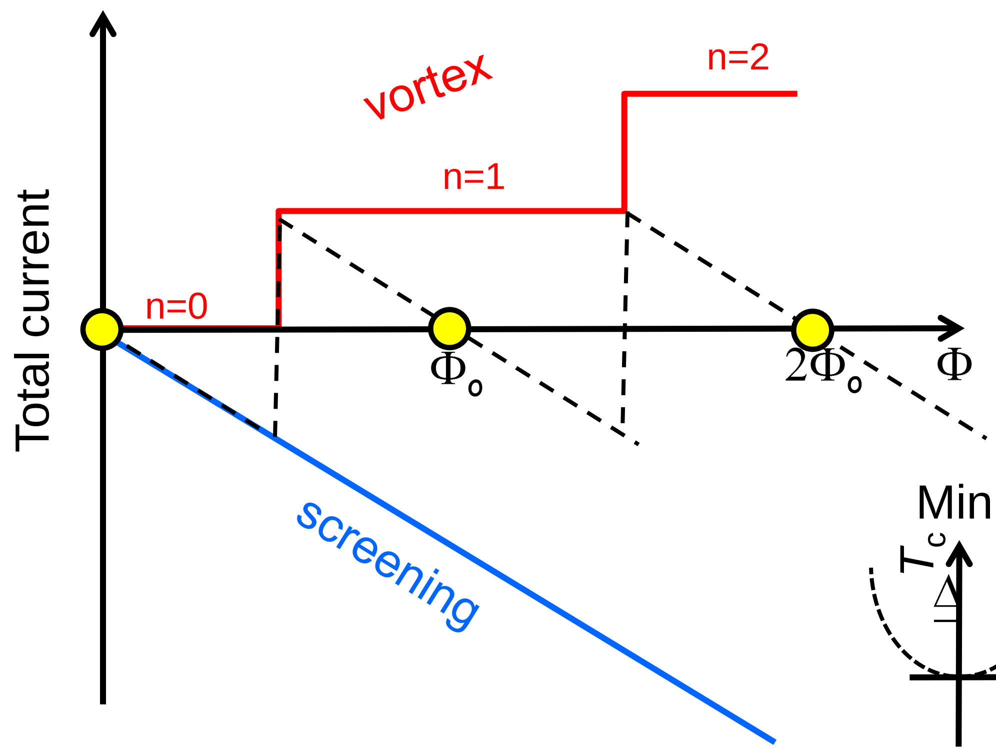
LITTLE – PARKS EXPERIMENT

W. A. Little and R. D. Parks, Physical Review Letters.**9**, 9 (1962).

$$\Phi + \frac{m}{2e^2 n_s} \oint J \cdot d\ell = n\Phi_o$$

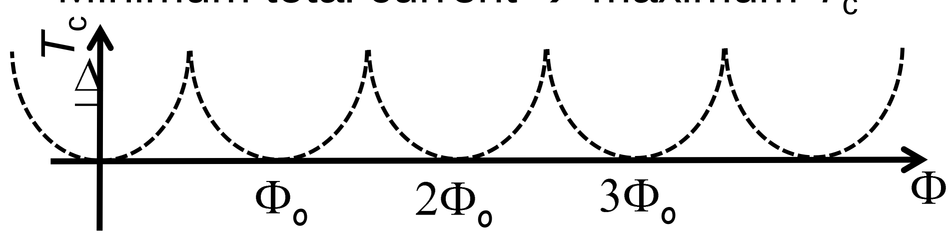
$\Phi = \Phi_{ext} + Li$





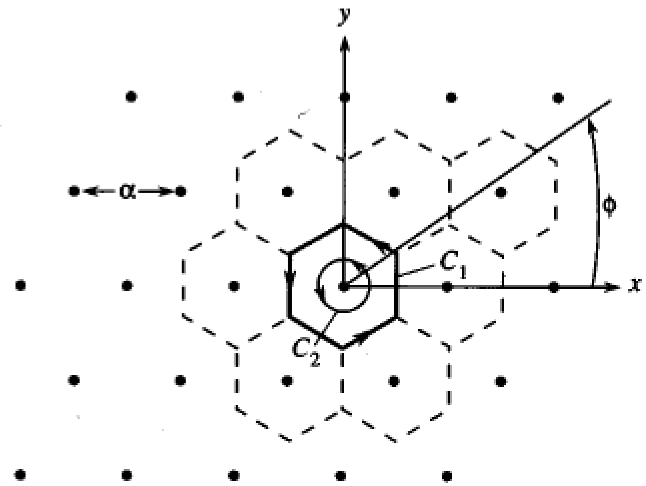
$\lambda \gg w \rightarrow \Phi + cJ = n\Phi_o$

Minimum total current → maximum T_c



A VORTEX IS A FLUXON ?

$$\Phi + \frac{mc}{e^*}\oint v_s \cdot dl = n\Phi_o$$



Fluxoid Quantization along C_1

$$n\Phi_o = \oint_{C_1} \mu_o \lambda^2 \mathbf{J}_S \cdot d\mathbf{l} + \int_{S_1} \mathbf{B} \cdot d\mathbf{s}$$

But along the hexagonal path C_1 **B** is a mininum, so that **J** vanishes along this path.

Therefore, $n\Phi_o = \int_{S_1} \mathbf{B} \cdot d\mathbf{s}$

And experiments give n = 1, so each vortex has one flux quantum associated with it.

Along path C_2, $\Phi_o = \oint_{C_2} \mu_o \lambda^2 \mathbf{J}_S \cdot d\mathbf{l} + \int_{S_2} \mathbf{B} \cdot d\mathbf{s}$

For small C_2, $\Phi_o = \lim_{r \to 0} \oint_{C_2} \mu_o \lambda^2 \mathbf{J}_S \cdot d\mathbf{l}$ ⇒ $\lim_{r \to 0} \mathbf{J}_S = \frac{\Phi_o}{2\pi \mu_o \lambda^2} \frac{1}{r} \mathbf{i}_\phi$

WHY A NORMAL CORE ?

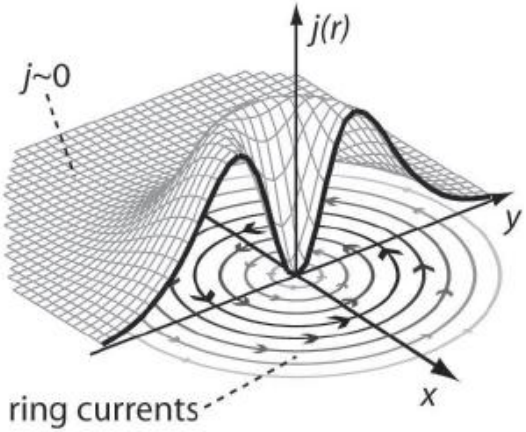
The current density $\lim_{r \to 0} \mathbf{J}_\mathrm{s} = \frac{\Phi_o}{2\pi\mu_o\lambda^2}\frac{1}{r}\mathbf{i}_\phi$ diverges near the vortex center,

Which would mean that the kinetic energy of the superelectrons would also diverge. So to prevent this, below some core radius ξ the electrons become normal. This happens when the increase in kinetic energy is of the order of the gap energy. The maximum current density is then

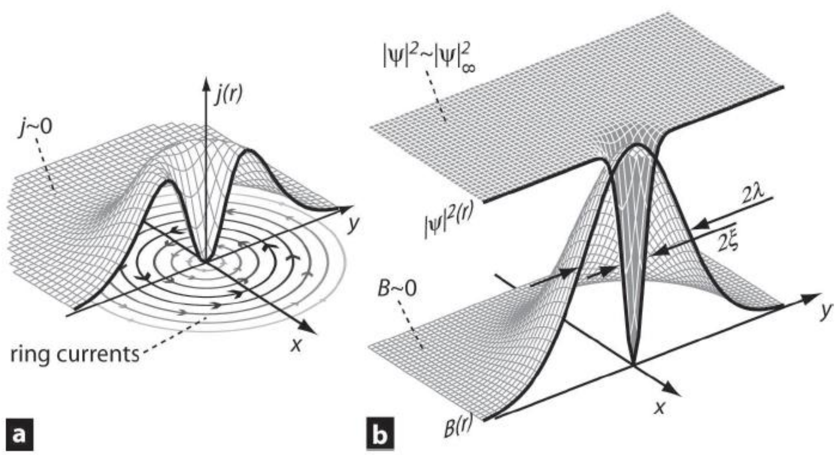
$$\mathbf{J}_\mathrm{s}^\mathrm{max} = \frac{\Phi_o}{2\pi\mu_o\lambda^2}\frac{1}{\xi}\mathbf{i}_\phi \quad \Rightarrow \quad \mathbf{v}_\mathrm{s}^\mathrm{max} = \frac{\hbar}{m^\star}\frac{1}{\xi}\mathbf{i}_\phi$$

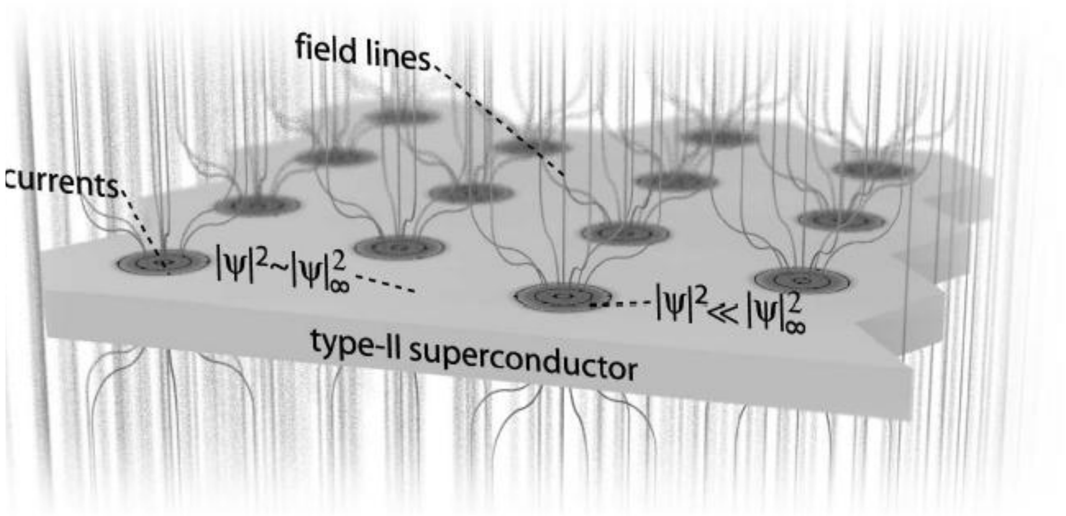
Therefore the maximum current density, known as the *depairing current density*, is

$$J_\mathrm{depair} \approx \frac{\Phi_o}{2\pi\mu_o\lambda^2\xi}$$



DEEP INTO THE SC STATE: VORTICES

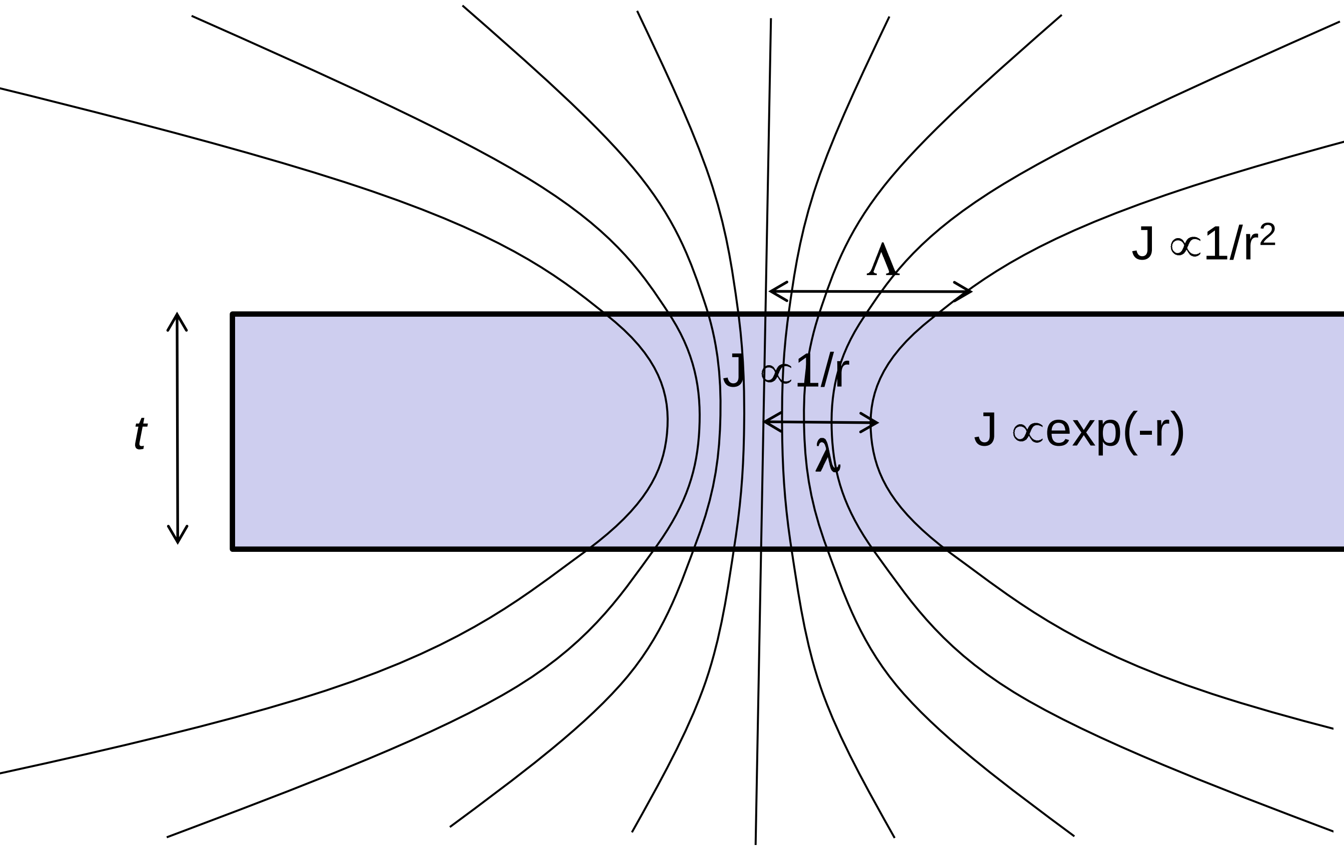




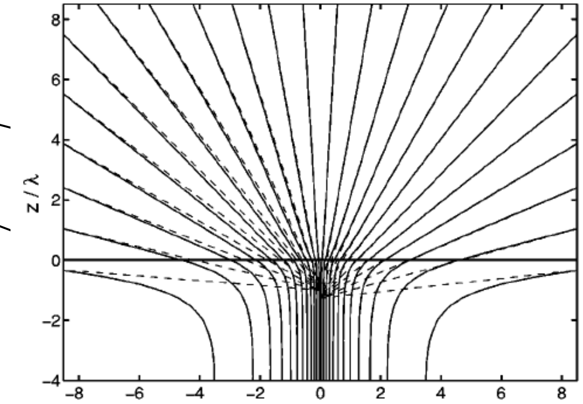
$$b(r) \to \frac{\Phi_o}{2\pi\lambda^2}\left(\frac{\pi\lambda}{2r}\right)^{1/2} e^{-r/\lambda} \qquad r \to \infty$$

$$b(0) \sim \frac{\Phi_o}{2\pi\lambda^2} = 2H_{c1}$$

VORTICES IN A THIN FILM



$$\Lambda = \left(\frac{\lambda^2}{t} \right)$$



J. Pearl, Appl. Phys. Lett. **5, 65 (1964)**

G. Carneiro and E.H. Brandt, Phys. Rev. B **61, 6370 (2000)**

VORTEX LINE ENERGY

$$\varepsilon = \varepsilon_{FIELD} + \varepsilon_{CURRENTS} + \varepsilon_{CORE}$$

$\varepsilon_{magnetic}$ $\qquad$ ε_{EM} (= $\varepsilon_{FIELD} + \varepsilon_{CURRENTS}$) $\qquad$ $\varepsilon_{kinematic}$

$$\varepsilon = \frac{1}{8\pi}\int B^2 dV + \int \frac{1}{2} m n_s v_s^2 + \frac{H_c^2}{8\pi}(\pi\xi^2)$$

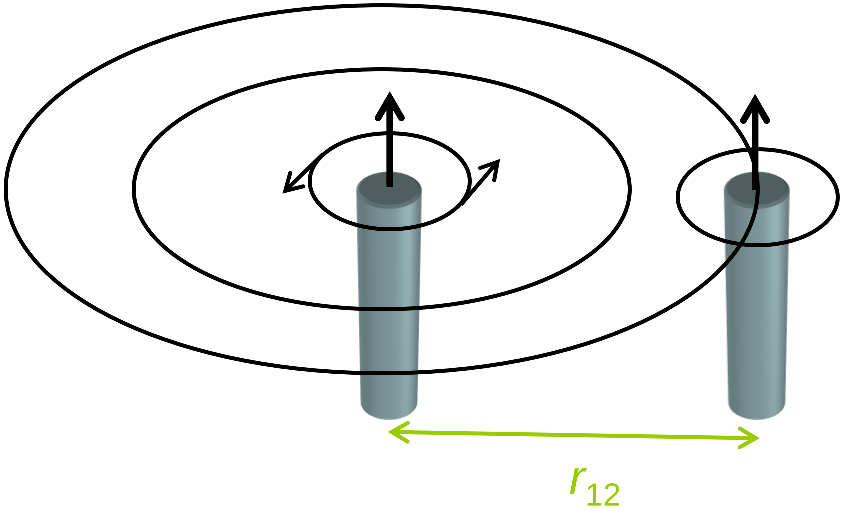
ASSUMING → AN INFINIT SC
→ κ>>1

$$\frac{\varepsilon_{kinematic}}{\varepsilon_{magnetic}} \approx 2(\ln\kappa + 0.115) \qquad \varepsilon_{EM} \sim (4\ln\kappa)\varepsilon_{CORE}$$

$$\varepsilon \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2\left(\ln\kappa + \frac{1}{4}\right) \qquad \varepsilon \sim \left(\frac{\Phi_o}{4\pi\lambda}\right)^2 \qquad \varepsilon \sim \frac{\Phi_o}{8\pi} b(0)$$

SELF-ENERGY

VORTEX-VORTEX INTERACTION



$$\varepsilon_T \sim 2\frac{\Phi_o}{8\pi}b(0) + 2\frac{\Phi_o}{8\pi}b(r_{12})$$

WE ARE IGNORING THE CORE !

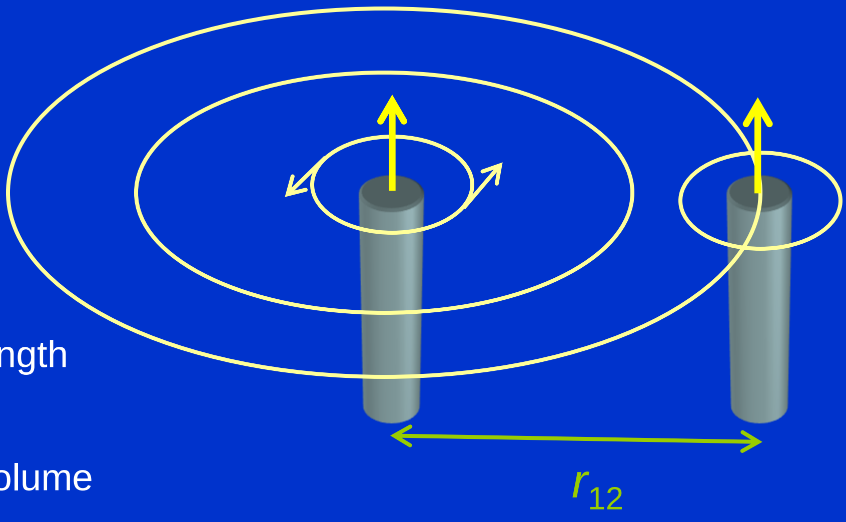
$$\varepsilon_{\text{int}} \sim \frac{2\Phi_o}{8\pi}b(r_{12})$$

$$f_{v-v} \sim -\frac{\partial \varepsilon_{\text{int}}}{\partial r} \sim \frac{\Phi_o}{4\pi}\frac{\partial b(r)}{\partial r}$$

$$\bar{f}_{v-v} = \bar{J}_s \times \overline{\Phi}_o$$

VORTICES REPEL EACH OTHER

VORTEX-CURRENTS INTERACTION



London Force

$$\bar{f}_L = \bar{J}_s \times \bar{\Phi}_o$$

Per unit length

$$\bar{f}_L = \bar{J}_s \times \bar{B}$$

Per unit volume

Has any thing to do the Lorentz force with the London force ?

A vortex also has a mass and a charge !

$$m^*\dot{\mathbf{v}} = -e^*\nabla\varphi + e^*\mathbf{v} \times \mathbf{B}$$

H. Suhl, Phys. Rev. Lett. 14, 226 (1965) J. Kolacek, et al., Phys. Rev. Lett. 86, 312 (2001)

CORE VS ELECTROMAGNETIC PINNING

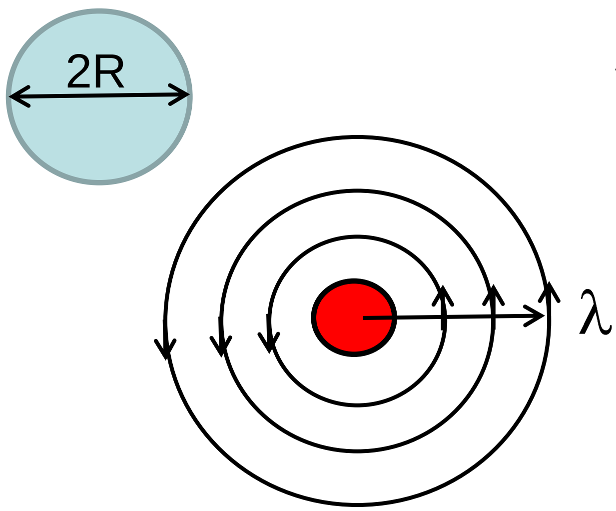
$$E_{OUT} = \left[\frac{\Phi_0}{4\pi\lambda}\right]^2 \left[\ln\left[\frac{\lambda}{\xi}\right] + c\right]$$

c contribution of the normal core (core pinning)

Cut-off of the current (electromag. pinning)

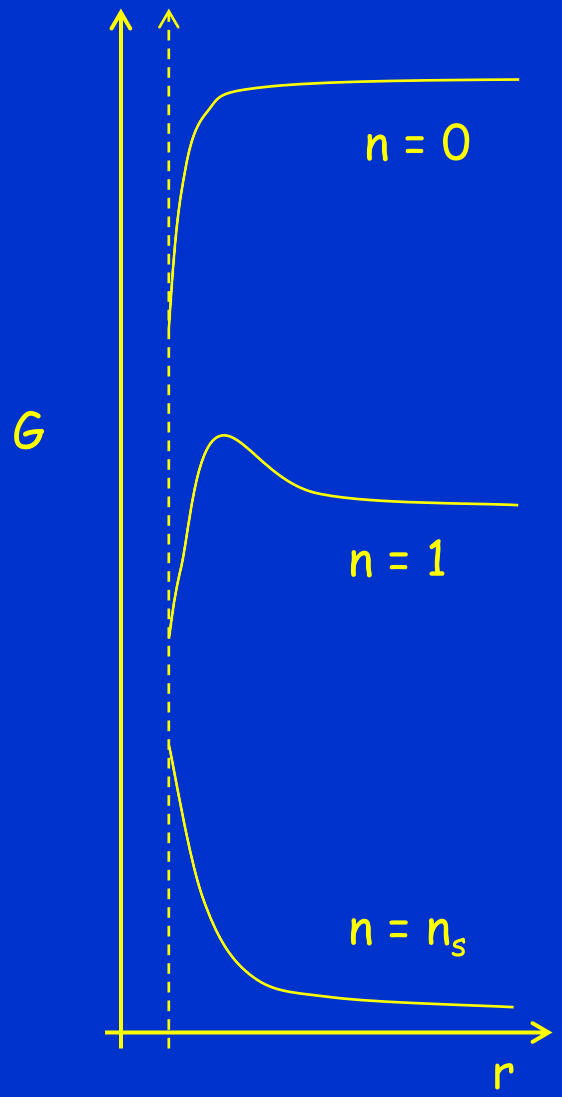
$$E_{IN} = \left[\frac{\Phi_0}{4\pi\lambda}\right]^2 \ln\left[\frac{\lambda}{R}\right]$$

$$E_{PIN} \equiv E_{IN} - E_{OUT} = -\left(\frac{\Phi_o}{2\pi\lambda}\right)^2 \left(\ln\frac{R}{\xi} + \frac{1}{4}\right)$$

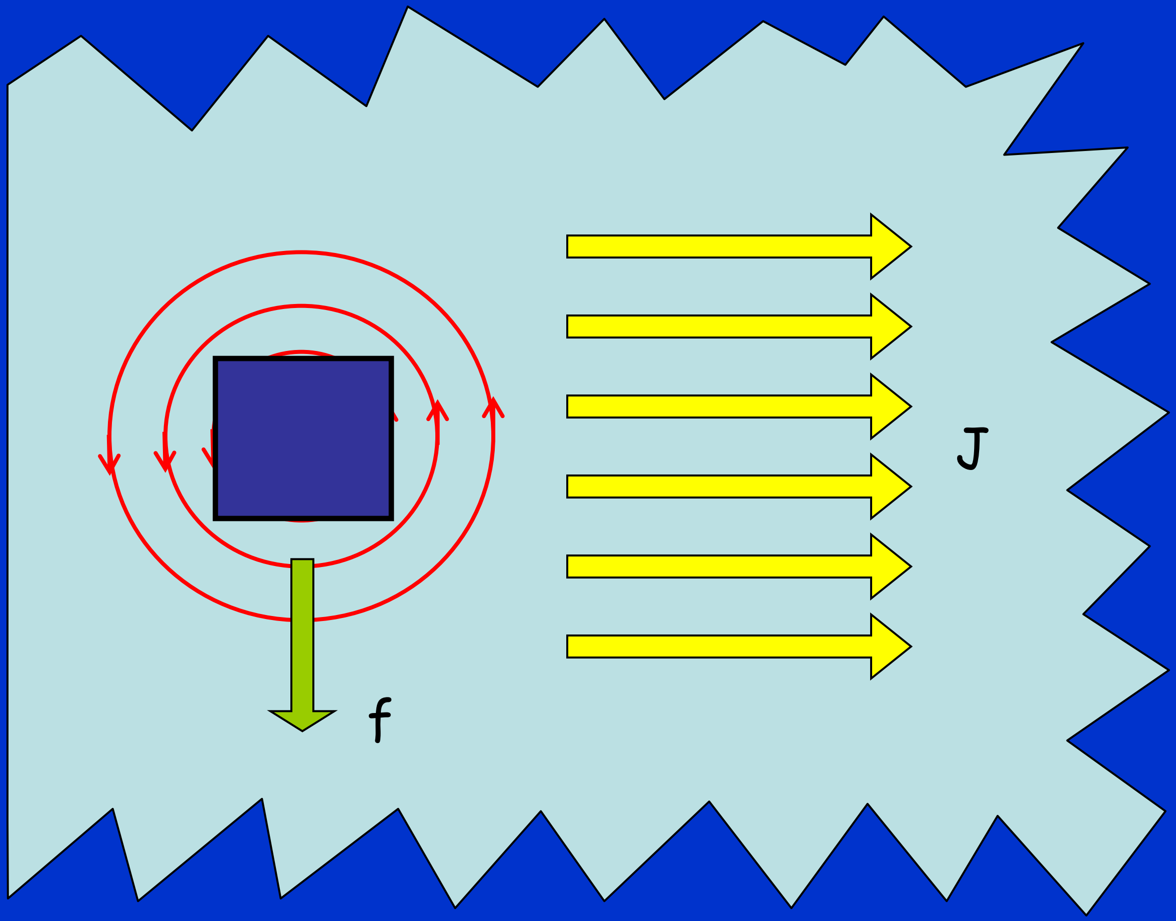


VORTICES CAN BE TRAPPED BYHOLES

MULTIQUANTA TRAPPING OF VORTICES



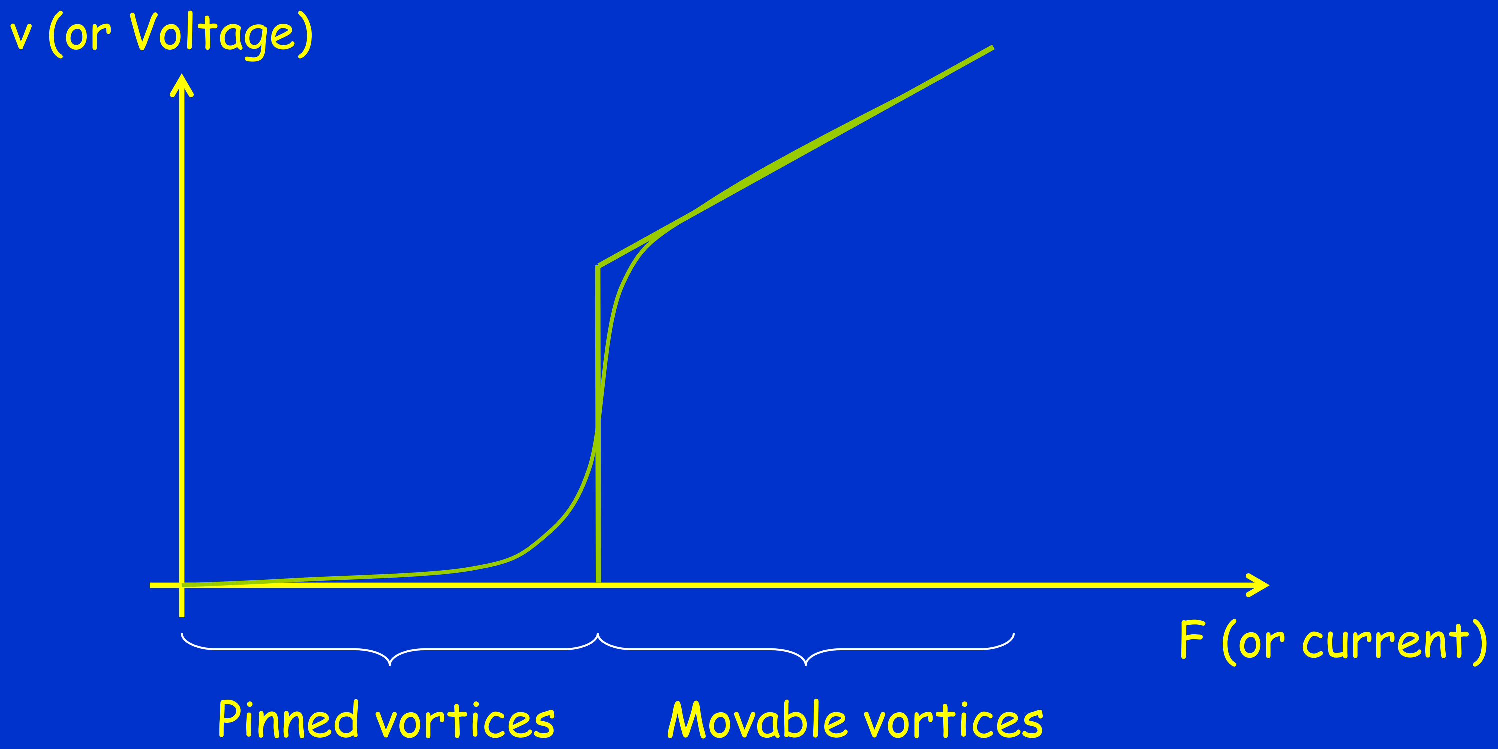
CRITICAL CURRENT



Jc = maximum current before depinning

Jo = maximum current before depairing ~ Hc/λ

V-F (or IV) CHARACTERISTICS

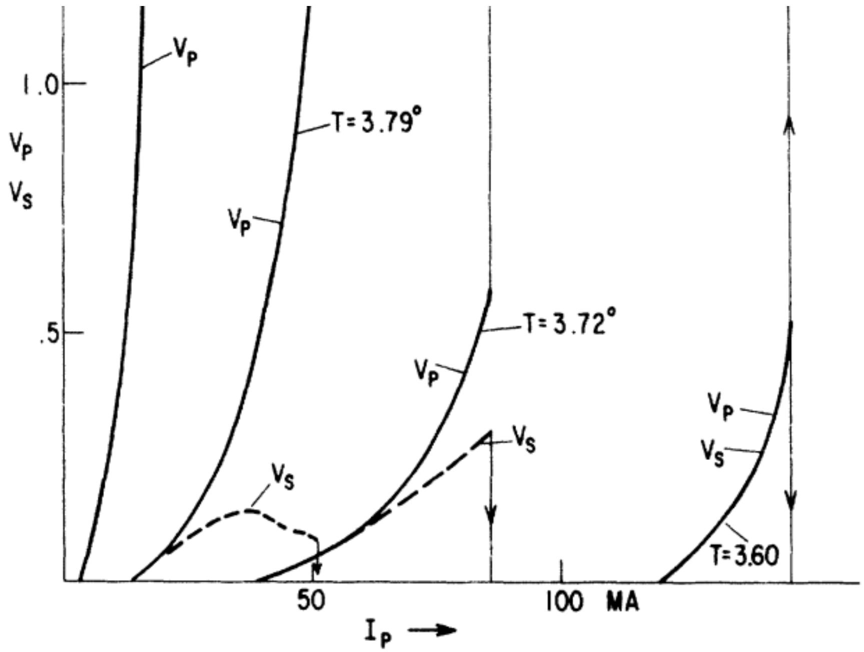


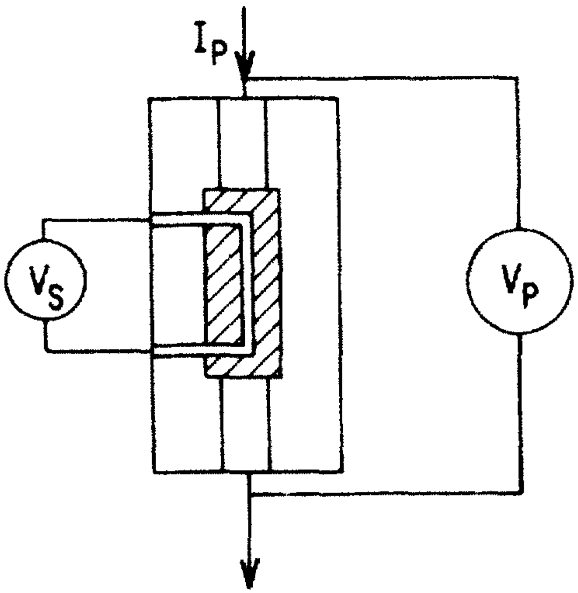
How fast a vortex can move ?

Andronov Physica C (1993)

Why a vortex in motion dissipates ?

EXPERIMENTAL EVIDENCE (Giaever)





ORIGIN OF DISSIPATION

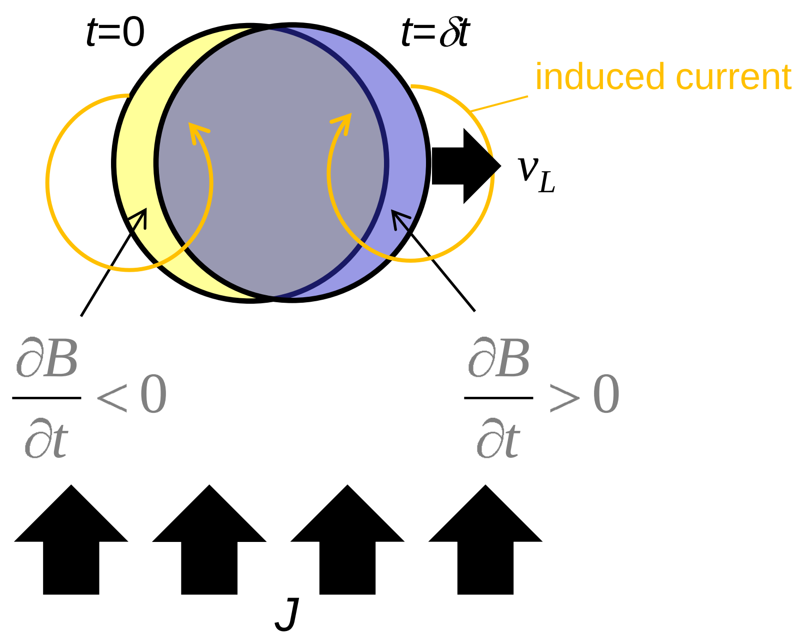
Faraday's law $\nabla \times E = -\frac{\partial B}{\partial t}$

Energy dissipated per unit volume and time

$$F_L v_L = Ej$$

$$F_L = jB$$

$$E = Bv_L$$

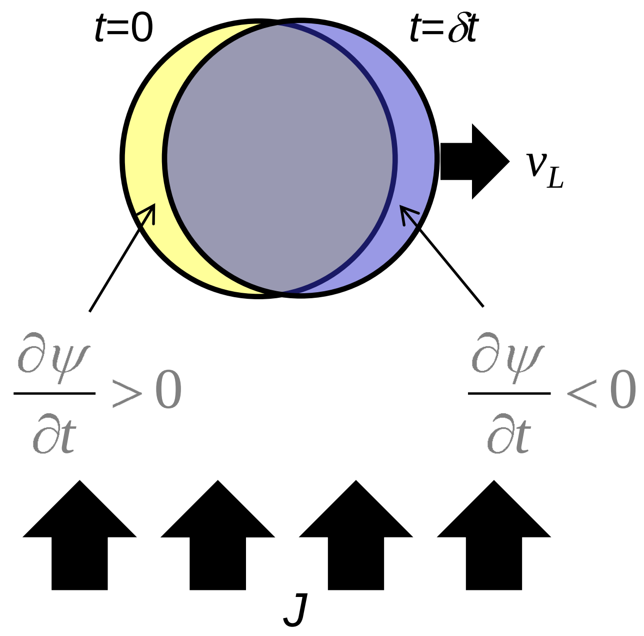


Induced currents oppose the change of flux (Lenz law)

damping

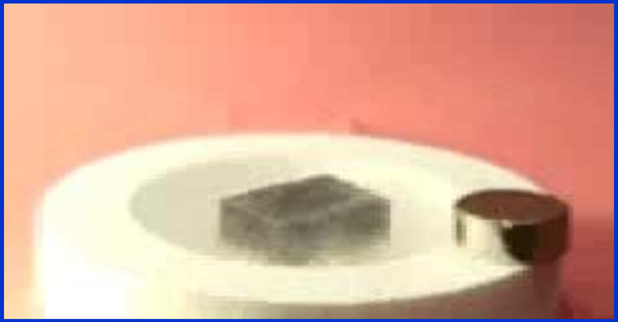
WE DO NOT NEED THE ORDER PARAMETER ?

$$F_s - F_n = -\frac{H_c^2}{8\pi}$$

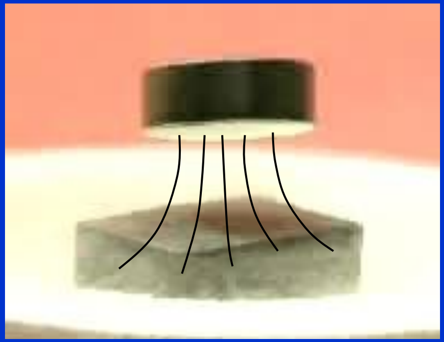


Condensation – destruction process also leads to dissipation

EXPERIMENTS

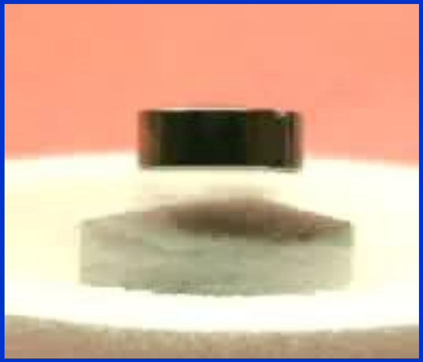


Meissner effect



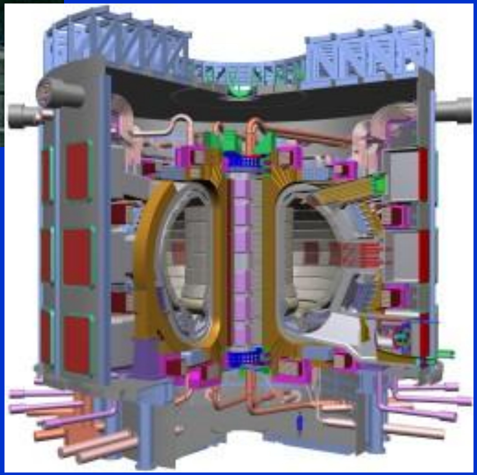
Vortex pinning (ZFC)

Vortex pinning (FC)

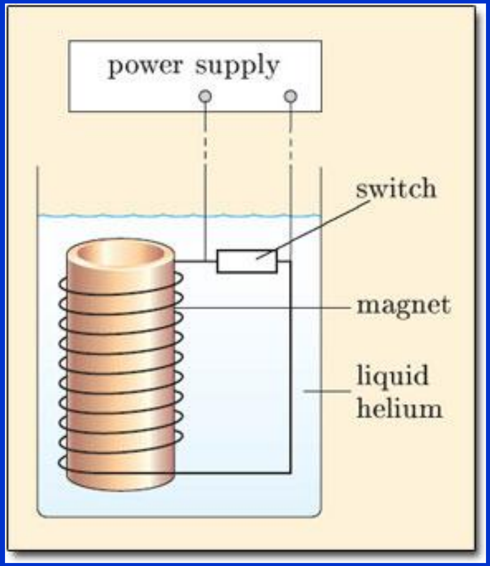


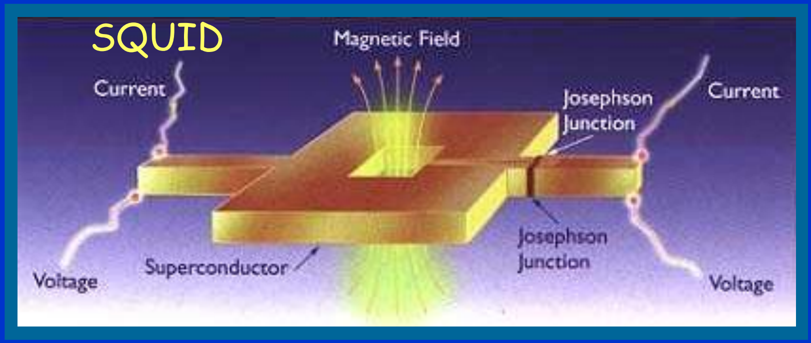
Existence of Tc

APPLICATIONS



International Thermonuclear Experimental Reactor (confinement)





"Physics is like sex. Sure, it may give some practical results, but that's not why we do it."
- R. Feynman (1918-1988)

CONCLUSION

- ✓ Hallmark of superconductivity → perfect conductor
  →perfect diamagnet

- ✓ There are two characteristic length scales $\xi \rightarrow \psi$
  $\lambda \rightarrow B$

- ✓ Both, ξ and λ, diverge at $T=T_c$ but are field independent.

- ✓ There are two species, Type I ($\lambda/\xi<1/\sqrt{2}$) → Normal domains $\phi \sim 1000\phi_0$
  Type II ($\lambda/\xi>1/\sqrt{2}$) → single vortices $\phi=\phi_0$

- ✓ Fluxoid is quantized

- ✓ Three theoretical formalisms, London (neglects core contributions, $\lambda/\xi>>1$)
  Ginzbug-Landau (close to T_c)
  BCS (weak coupling)

- ✓ Superconductivity likes to nucleate at the surfaces (if insulating)
- ✓ Vortices, repel each other (Abrikosov lattice), move perpendicular to the current flow dissipate when moving, their motion can be avoided by introducing pinning centers.

THE END